VLBI Clock Sync and the Earth's Rotational Instability

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The DSN is currently preparing to monitor the stability of its clocks and frequency standards in the 64-meter net by means of VLBI. Since variations in the Earth's rotation rate represent an error source to VLBI clock synchronization, we calculated the Allan Variance of the Earth rotation to find that, in a long-term sense at least, these variations do not noticeably increase the differential instability of two clocks as measured by Intercontinental VLBI.

I. Introduction

The DSN is currently preparing to monitor the stability of its clocks and frequency standards in the 64-meter net by means of VLBI. Since variations in the Earth's rotation rate represent an error source to VLBI clock synchronization, it is not immediately clear that such frequency standards monitoring is possible to the precision desired. We calculated the Allan Variance of the Earth rotation to find that, in a long-term sense at least, the instability of the Earth's rotation does not cause the differential instability of two widely separated clocks as measured by Intercontinental VLBI to measurably exceed that intrinsic to the clocks themselves, which is on the order of one part in 10^{14}.

II. The VLBI Time Delay

Roughly, the time delay difference measured in a VLBI experiment for any particular radio source observed at UTC (Universal Time-Coordinated) = t is given by (Ref. 1)

$$\Delta Tm(t) = \Delta Tc(t) + Z_b \sin \delta_s$$

$$+ r_b \cos \delta_s \cos (\alpha_s - \lambda_b - a_G(t)) \qquad (1)$$

$$- (UT_1 - UTC)) + \Delta T_i(t)$$

The various elements of Eq. (1) are defined as follows:

$\Delta Tc(t)$ the difference between clocks at the stations

Z_b the projection of the baseline between the two stations upon the instantaneous spin axis of the Earth

r_b the projection of the baseline into the instantaneous equatorial plane of the Earth

λ_b the longitude of the baseline

$a_G(t)$ Greenwich Siderial Time at $UTC = t$

α_s and δ_s the right ascension and declination of the radio source

$(UT_1 - UTC)$ the deviation in the phase of the Earth's rotation relative to UTC, measured in radians of Earth rotation

$\Delta T_i(t)$ the difference in incidental time delays such as propagation medium and instrumentation

Those elements of Eq. (1) of principal interest here are the clock differences $\Delta Tc(t)$, and $(UT_1 - UTC)$. The equation is clearly nonlinear in the $(UT_1 - UTC)$ parameter for large

excursions, but is linear for small ones with a coefficient on the order of 2 μs of time delay per second of UT_1 for an individual delay measurement. The effective coefficient can be somewhat reduced for slow variations in $(UT_1 - UTC)$ by solving for $(UT_1 - UTC)$ using observations of a number of appropriately located radio sources, but fast variations can masquerade equally well as fast variations in the clock differences irrespective of such techniques.

III. Instability Statistic

The Allan Variance (Refs. 2 and 3) is routinely used to measure instability of an oscillator. Let

$$y(t) = a \sin \left(2\pi\nu_0 t + \phi(t)\right) \tag{2}$$

be the signal emitted by the oscillator at nominal frequency ν_0. The two-sample Allan Variance with measurement interval T may be computed as

$$\sigma_\phi^2(T) = \frac{1}{2N} \sum_{i=1}^{N} \left[\frac{\phi(t_i) - 2\phi(t_i + T) + \phi(t_i + 2T)}{2\pi\nu_0 T}\right]^2 \tag{3}$$

The square-root of this statistic is an estimate of the fractional error in time as counted by this oscillator, or of the fractional error in oscillator frequency. The instability measure of the DSN Hydrogen Maser is essentially constant at 5×10^{-15} for $T > 30$ sec (Ref. 4, Fig. 3).

The Earth itself is an oscillator which operates at a nominal rate of one cycle per "day." Its instabilities take two forms: motion of the poles or spin axis relative to the Earth's crust, and changes in the rotation rate about the instantaneous position of the spin axis. Motion of the spin axis is measured in a rectangular coordinate system (X, Y) centered at the mean pole of 1903; variations in the phase of rotation appear in $(UT_1 - UTC)$. Polar motion is implicit in Eq. (1) in the definition of Z_b, r_b relative to the instantaneous spin axis. Our data for X, Y, and $(UT_1 - UTC)$ comes via the US Naval Observatory Time and Frequency Bulletins, Series 7, and is computed by the Bureau International de l'Heure (BIH) based on optical observations by a world-wide observatory network. The so-called "Rapid Service" values are published for one-day intervals within a week of the actual date, and can be expected to be somewhat noisier than the "Final Values," which are published for five-day intervals after a month of additional observing.

Figure 1 shows the fractional instability of the Earth's rotation as computed from the Rapid Service data for the interval from January 1976 to July 1977. For comparison, the polar motions X and Y are scaled in fractional Earth rotations (days), as is $(UT_1 - UTC)$. The region of UT_1 for $T < 3$ days, which decreases as $1/T$, is representative of a white noise error on individual determinations, and is of a magnitude which is at least conceptually consistent with the 4-ms error (Ref. 5) attained by individual observatories in one night. The peak at around 70 days could be from the seasonal variations in UT_1, which are known to exist but hard to predict.

Figure 2 shows the fractional instability of the Earth's rotation as computed from the Final Values for the same interval. Statistically, the difference between Rapid Service and Final Values is that all of the apparent white noise of observations is removed in the Final Values, along with most short-term variations.

Figure 3 shows the fractional instability of the Rapid Service UT_1 from February 1972 through June 1977. The 1-sigma confidence intervals shown correspond to the number of distinct T-day segments in the data record. For $10 < T < 100$ days, this figure is essentially the same as Fig. 1. For $T > 200$ days, $\sigma(\Delta T/T)$ is approximately constant, possibly representing a "flicker" noise component of 2×10^{-9}. This is consistent with the observation in Ref. 6 that variations in $UT_2 - A.3$ during 1956-1965 were flicker-like.

We should be cautious in obtaining conclusions based upon these figures for two reasons. First of all, the data set we have analyzed is short relative to some of the known variations. Secondly, we are at the mercy of the instrumentation and averaging methods used by the BIH, so that the estimated instability will include some instrumental noise, and it could be missing some real variations due to the averaging. Thus, if conclusions are to be formed from this data, a sizeable margin should exist relative to the calculated statistics.

IV. Implication for VLBI Clock Sync

As noted earlier, errors in UT_1 could masquerade as clock errors in VLBI time delay measurements to the level of at worst 2 μs clock error per second of UT_1 error. The fractional instability of the VLBI-measured clock contains terms due to UT_1 variations and to the clocks themselves. These terms are shown in Fig. 4. If we assume that the BIH Final Values represent the actual Earth motion, then the noise induced in a VLBI-measured clock by use of the Rapid Service data corresponds to the noise of the difference: UT_1 (Rapid Service) - UT_1 (Final). For short time intervals, such as a single VLBI observing pass, we cannot tell absolutely whether short-term variations in UT_1 represent a problem or not, because we do not know whether the white phase noise in

Rapid Service UT_1 for $T < 3$ days is really in UT_1 or is an artifact of the optical instruments currently used to measure it. This matter could be resolved by attempting to use VLBI to solve for UT_1 on a shorter time interval than can be done with the optical data.

For time scales in excess of 3 days, it appears in Fig. 4 that instability of the frequency standards themselves is comparable to or greater than even the worst-case statistical variations due to the Rapid Service data. Both are below the 10^{-14} level, and thus the Earth's rotational instability should cause no problem in monitoring the frequency standards' long-term performance to a few parts in 10^{14}, even without concurrently solving for UT_1.

References

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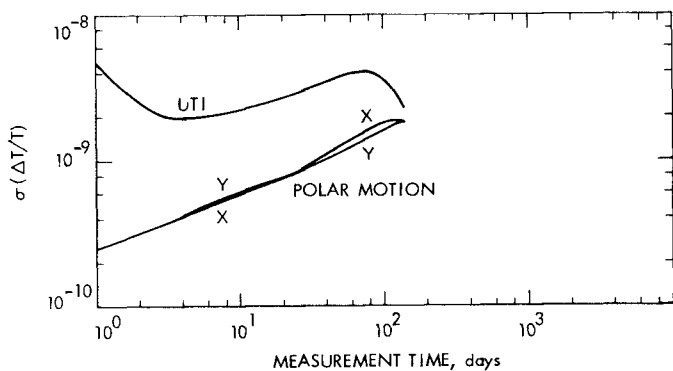
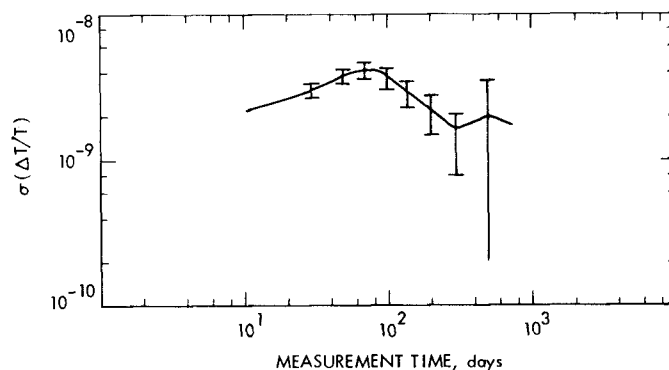**Fig. 1. Stability of Earth rotation, BIH Rapid Service data, January 1976 to June 1977**

Fig. 2. Stability of Earth rotation, BIH Final Data, January 1976 to June 1977

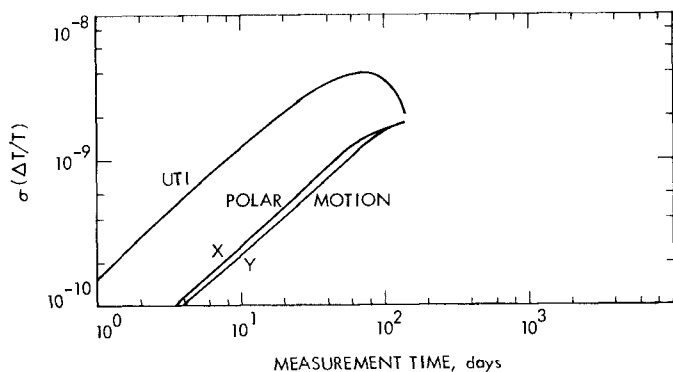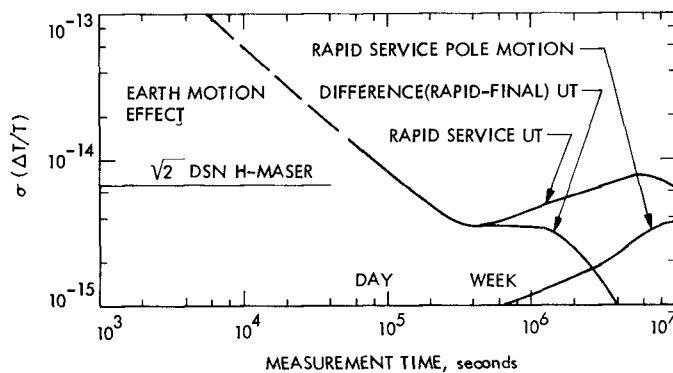**Fig. 3. Stability of Earth rotation, BIH data, February 1972 to June 1977**

Fig. 4. VLBI-measured clock rate variations